

MATHEMATICAL GEOMORPHOLOGY

Preface

The present work is an abbreviation of “Computer Geomorphology” in Japanese, centering mainly around figures and basic equations used for digital simulations. Landforms are made and transformed by the interaction of earth surface materials and geomorphic agents such as running water, wind, glacier and so on. These are physical, mainly mechanical processes which can be and must be explained by the principles of physical sciences.

The purpose of the present work is to clarify the physical processes by means of reproducing major landforms numerically in a computer. Basic equations used for the simulation are limited to those which have sound physical bases. The validity of the numerically reproduced landforms is confirmed by morphological properties of real and experimental landforms.

1. Landform analysis

1.2 Basic equations

Landforms are formed and transformed as the results of the movement of earth surface materials. The processes are described by the equation of continuity of the materials.

In Fig.1.1 the following relation holds:

$$MB\Delta t - \left(MB + \frac{\partial(MB)}{\partial x} \Delta x \right) \Delta t + M_s \Delta x \Delta t = \Delta V = \frac{\partial z}{\partial t} \Delta t B \Delta x (1 - \lambda) \quad (1.1)$$

where M is mass flux, B is width of layer, t is time, x is horizontal distance, M_s is lateral inflow of M , V is the volume, z is vertical distance and λ is porosity of deposits layer.

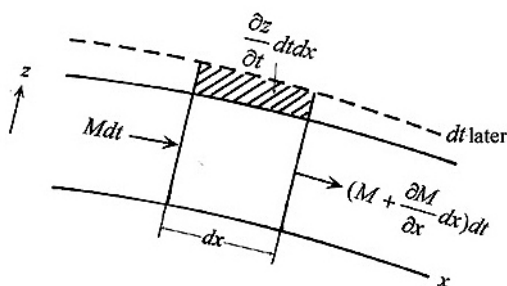


Fig.1.1
Condition of continuity

By arranging the relation, continuity equation in one-dimension is derived:

$$\frac{\partial z}{\partial t} + \frac{1}{(1 - \lambda)B} \left(\frac{\partial(MB)}{\partial x} - M_s \right) = 0 \quad (1.2)$$

A simplified form is

$$\frac{\partial z}{\partial t} - \frac{\partial M}{\partial x} = 0 \quad (1.3)$$

It is mass flux M that creates characteristic landforms for each geomorphic agents. This is mainly given by a power function of geomorphic force. For example, tractional load by running water is a function of about 3rd power of tractive force, and erosion rate by the movement of glacier is proportional to 2nd power of basal sliding velocity. To obtain these geomorphic forces, equation of motion and equation of continuity for running water, wind, wave, glacier and so on are needed.

Running water on earth surface is the most dominating geomorphic agent. Equation of motion and equation of continuity in two-dimensions are as follows:

$$\frac{\partial M}{\partial t} + u \frac{\partial M}{\partial x} + v \frac{\partial M}{\partial y} = -gh \frac{\partial(h+z)}{\partial x} - \frac{\tau_{bx}}{\rho} \quad (1.4)$$

$$\frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = -gh \frac{\partial(h+z)}{\partial y} - \frac{\tau_{by}}{\rho} \quad (1.5)$$

$$\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = r \quad (1.6)$$

$$\tau_x = \rho g n^2 u |u| / h^{1/3} \quad (1.7)$$

$$\tau_y = \rho g n^2 v |v| / h^{1/3} \quad (1.8)$$

where M is discharge to x -direction, N is discharge to y -direction, u is flow velocity to x -direction, v is flow velocity to y -direction, h is water depth, z is ground height, g is gravity acceleration, ρ is density of water, r is lateral inflow by rainfall and n is Manning's roughness coefficient. Erosive force of running water is obtained from these equations.

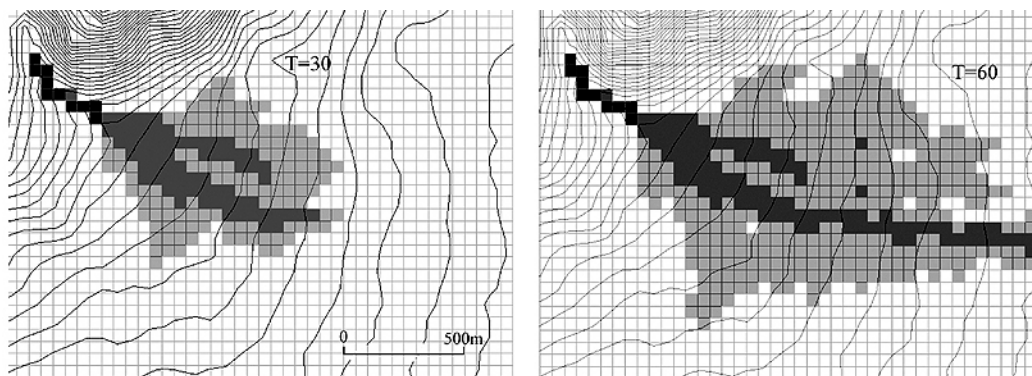


Fig.1.9 Example of two-dimensional computer simulation of river flooding on fan surface